Type I supergravity effective action from pure spinor formalism

JHEP02(2009)025

(http://iopscience.iop.org/1126-6708/2009/02/025)

Published by IOP Publishing for SISSA
Received: *January 1, 2009*
Accepted: *January 21, 2009*
Published: *February 9, 2009*

Type I supergravity effective action from pure spinor formalism

Geová Alencar

Instituto de física teórica, Universidade Estadual Paulista,
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil

Abstract: Using the pure spinor formalism, we compute the tree-level correlation functions for three strings, one closed and two open, in $N = 1$ $D = 10$ superspace. Expanding the superfields in components, the respective terms of the effective action for the type I supergravity are obtained. All terms found agree with the effective action known in the literature. This result gives one more consistency test for the pure spinor formalism.

Keywords: Superstrings and Heterotic Strings, BRST Quantization, Topological Strings, Supersymmetric Effective Theories.

JHEP02(2009)025

Contents

JHEP02(2009)025

1. Introduction

The covariant quantization of superstring theory has been an unresolved problem for a long time. The covariant quantization, besides having manifest supersymmetry, makes the computation of scattering amplitudes easier. This is important for understanding the low energy limit of superstrings, through the construction of effective actions corresponding to such amplitudes. In order to solve the problem of manifest covariant quantization, a new formalism, known as pure spinors formalism, was proposed [1]. This new formalism keeps all the good properties of Ramond-Neveu-Schwarz and Green-Schwarz and does not have its undesired characteristics. In the Ramond-Neveu-Schwarz formalism, when the number of loops in computations of scattering amplitudes are increased, more and more spin structures have to be considered which makes the computations very long. On the other hand, in the Green-Schwarz formalism the quantization is only possible in the light-cone gauge and the amplitude computations involve non-covariant operators at the interaction points.

The complete equivalence between the pure spinor formalism and other formalisms is missing. Until this point is reached, the formalism needs to pass many consistency tests. One of these tests consists of computing scattering amplitudes and comparing the results with those coming from other formalisms. These tests have been carried out for amplitudes involving closed superstrings at one loop [2] and two loops [3], among others. Interestingly, amplitudes involving mixed superstrings have not been considered in the literature. One important point is the fact that in pure spinor formalism the amplitudes have explicit super-Poincaré symmetry, making the results automatically supersymmetric since the beginning.

From the viewpoint of field theory, the effective action for the type I supergravity is obtained from the global super Yang-Mills action by imposing local supersymmetry, which originates many compensation terms [4] that are interpreted as interaction terms. From the viewpoint of superstrings, all these interaction terms must come out naturally from amplitude computations. The interaction terms of the type I supergravity effective action have some interesting properties. A very peculiar one is the fact that there is a coupling between the Kalb-Ramond and two photons. This term is needed to obtain local supersymmetry. In order to keep gauge invariance, the Kalb-Ramond field must have a unusual transformation under U(1) symmetry. This coupling will become very important for the mixed anomaly cancelation in the SO(32) theory.

As pointed above, all these terms must come naturally from superstring theory. However, as they involve gravitational and Yang-Mills fields, amplitudes with open and closed strings have to be considered. In this work, we will show that these terms can be obtained from the pure spinor formalism. In string theory, the effective action can be obtained considering the scattering amplitudes or correlation functions of three points given by

$$\mathcal{A} = \langle V_1 V_2 V_3 \rangle,$$

where the Vs above represent the physical states and are called vertex operators. For the mixed scatterings, we must use the upper-half complex plane. The closed string is represented by a point in the interior of the plane, while the open strings, by points in the real axis. The number of conformal Killing vectors in this case is three, and the number of moduli is zero. This makes it possible to fix the positions of the closed string and of one of the open strings, obtaining

$$\mathcal{A} = \langle V_1 V_2 \int U_3 \rangle.$$

In the last equation, V_1 represents the fixed closed string, V_2 represents the fixed open string and U_3, the integrated open string.

In the second section of this paper, we give the expression for the type I supergravity effective action and its respective linearized action. This will be compared with the expression obtained from the pure spinor formalism. In the third section, the prescription for amplitudes in pure spinor formalism will be briefly summarized, and an expression for the computation of one closed and two open strings will be given. Using superspace identities, a simple expression will be found following identical steps of [5]. This expression is given by

$$\mathcal{A} = g_o'^2 g_c' \pi i \alpha' \left\langle \left[A_m^1 \left(\lambda \tilde{A}^1 \right) + \tilde{A}_m^1 \left(\lambda A^1 \right) \right] \left(\lambda A^2 \right) \left(\lambda \gamma^m W^3 \right) \right\rangle,$$

where the superfields in the closed string vertex operator V^1 have been written as the product of two open string superfields.

It is important to note that this expression is manifestly super-Poincaré covariant and that all the amplitudes involving one closed and two open massless strings are contained in it. This shows one of the advantages of the pure spinor formalism. It will be also shown, in appendix B, that this expression has all the gauge invariances.

In the fourth section, with this simple expression, the superfields will be expanded in components, and explicit results will be given for all the correlation functions from which the effective action can be obtained. All cases will be considered with some details, except the case of one gravitino, one photon and one photino, which will be left for appendix D.

In appendix A, some useful identities will be given, and in appendix C, the correlation function for graviton-photon-photon will be considered in the Ramond-Neveu-Schwarz formalism for matter of comparison with pure spinor formalism.

2. Type I supergravity effective action

As said before, the Type I effective action can be obtained by imposing local supersymmetry in the Super-Maxwell action and is given by [4]

$$\begin{aligned}\frac{1}{\sqrt{-G}}\mathcal{L} = & -\frac{1}{2\kappa'}\tilde{R} - \frac{9}{16\kappa'^2}\left(\frac{\partial_m\phi}{\phi}\right)^2 - \frac{1}{4}\phi^{-3/4}\left(F_{mn}\right)^2 - \frac{3}{4}\phi^{-3/2}H^2_{mnp} \\ & -\frac{1}{2}\psi_m\gamma^{mnp}D_n\psi_p - \frac{1}{2}\lambda\gamma^m D_m\lambda - \frac{1}{2}\chi\gamma^m D_m\chi \\ & -\frac{1}{4}\kappa'\phi^{-3/8}\chi\gamma^m\gamma^{np}F_{np}\left(\psi_m + \frac{\sqrt{2}}{12}\gamma_m\lambda\right) \\ & +\frac{\sqrt{2}}{16}\kappa'\phi^{-3/4}\chi\gamma^{mnp}\chi H_{mnp} - \frac{3\sqrt{2}}{8}\psi_m\gamma^n\gamma^m\lambda\left(\frac{\partial_n\phi}{\phi}\right) \\ & +\frac{\sqrt{2}\kappa'}{16}\phi^{-3/4}H_{npq}(\psi_m\gamma^{mnpqr}\psi_r + 6\psi^n\gamma^p\psi^q \\ & -\sqrt{2}\psi_m\gamma^{npq}\gamma^m\lambda) + (Fermi)^4\,.\end{aligned}$$

In order to go to the String Frame, we make the field redefinitions

$$\tilde{G}_{mn} = e^{2\omega}G_{mn}, \qquad \phi = e^{8\omega/3}, \qquad \psi_m = \frac{1}{\kappa'}e^{\omega/2}\psi'_m, \qquad \chi = \frac{1}{g'}e^{-5\omega/2}\xi$$

$$\lambda = \frac{1}{\kappa'}e^{-\omega/2}\lambda', \qquad A_m = \frac{1}{g'}A'_m, \qquad B_{mn} = \frac{\kappa'}{3\sqrt{2}g'^2}B'_{mn}, \qquad \eta = e^{\omega/2}\eta',$$

where $\omega = (\Phi_0 - \Phi)/4$. We obtain the following Lagrangian

$$\begin{aligned}\mathcal{L} = & \frac{\sqrt{-G}}{2\kappa^2}e^{-2\Phi}\left[R + 4\partial_m\Phi\partial^m\Phi\right] - \frac{\sqrt{-G}}{4\kappa^2}e^{-\Phi}F^2_{mo} - \frac{1}{24\kappa^2}\sqrt{-G}H'^2_{mnp} \\ & -\frac{1}{2\kappa^2}\sqrt{-G}e^{-2\Phi}\psi_m\gamma^{mnp}D_n\psi_p - \frac{1}{\kappa^2}\sqrt{-G}e^{-2\Phi}\lambda\gamma^m D_m\lambda \\ & -\frac{1}{2\kappa^2}\sqrt{-G}e^{-\Phi}\xi\gamma^m D_m\xi - \frac{1}{4\kappa^2}\sqrt{-G}e^{-\Phi}\xi\gamma^m\gamma^{np}F_{np}\left(\psi_m + \frac{1}{6}\gamma_m\lambda\right)\end{aligned}$$

$$+\frac{1}{48}\frac{1}{\kappa^2}\sqrt{-G}\xi\gamma^{mnp}\xi H'_{mnp} - \frac{2}{\kappa^2}\sqrt{-G}e^{-2\Phi}\psi_m\gamma^n\gamma^m\lambda\partial_n\Phi$$
$$+\frac{3}{8\kappa^2}\sqrt{-G}e^{-\Phi}H'_{npq}(\psi_m\gamma^{mnpqr}\psi_r + 6\psi^n\gamma^p\psi^q - 2\psi_m\gamma^{npq}\gamma^m\lambda).$$

In the last equation, ψ is the gravitino, λ is the dilatino, ξ is the photino and we use the standard notation for the bosonic fields. The coupling between the Kalb-Ramond field and the photon comes from H'^{mnp}, which is defined as

$$H' = dB + 3AdA.$$

In the low energy limit, we need only the linearized Lagrangian. The usual procedure is to make

$$G^{mn} = \eta^{mn} + h^{mn}.$$

In order to simplify some terms, we use the identities

$$\gamma^m\gamma^{no} - \gamma^{no}\gamma^m = -2G^{mo}\gamma^n + 2G^{mn}\gamma^o$$

and

$$\gamma^m\gamma_{np}\gamma_m = 6\gamma_{np}.$$

Regarding only the terms related to the two open and one closed string amplitudes, we obtain

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2\kappa^2}h^m_n F_{mo}F^{no} - \frac{\Phi}{4\kappa^2}F^2_{mo} - \frac{1}{8\kappa^2}H_{mnp}A^m F^{np} \\ &-\frac{1}{2\kappa^2}h_{mn}\xi\gamma^m\partial^n\xi + \frac{1}{48}\frac{1}{\kappa^2}\xi\gamma^{mnp}\xi H_{mnp} \\ &-\frac{1}{\kappa^2}\xi\gamma_p\psi_m F^{mp} - \frac{1}{4\kappa^2}\xi\gamma^{np}\lambda F_{np}.\end{aligned} \tag{2.1}$$

The indices here are raised with η. Attention is required in considering the dilaton contribution coming from the expansion of the metric determinant, because

$$\sqrt{-G} = 1 + \frac{1}{2}h^m_m$$

and the trace of h_{mn} is related to the dilaton.

3. The tree-level correlation function for one closed and two open massless strings in the pure spinor formalism

As discussed in the introduction, from the viewpoint of string theory, the graviton is represented by a closed string and the photon by an open string. In pure spinor formalism, the fixed operator for the open string is given by [1]

$$V = g'_o\lambda^\alpha A_\alpha,$$

where λ is a pure spinor satisfying

$$\lambda\gamma^m\lambda = 0. \tag{3.1}$$

JHEP02(2009)025

The BRST operator is given by

$$Q = \lambda^\alpha d_\alpha$$

with

$$d_\alpha = \frac{\alpha'}{2} p_\alpha - \frac{1}{2}\theta\gamma^m \partial x_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\delta\eta}\theta^\beta\theta^\delta\partial\theta^\eta .$$

The physical state condition gives us the equations of motion

$$D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m, \tag{3.2}$$

where

$$D_\alpha = \frac{\alpha'}{2}\partial_\alpha + \theta\gamma^m\partial_m, \quad \bar{D}_\alpha = \frac{\alpha'}{2}\bar{\partial}_\alpha + \bar{\theta}\gamma^m\partial_m$$

and A_m is a vector superfield. The integrated vertex operator for the open string is

$$g'_o \int dy_3 \left(\partial\theta^\alpha A_\alpha + A_m \Pi^m + d_\alpha W^\alpha + \frac{1}{2} N^{nm}\mathcal{F}_{nm}\right),$$

where A_α, A_m and d_α are defined above and W^α and F_{mn} are field strengths given by

$$W^\alpha = \frac{1}{10}\gamma^{\alpha\beta}_m D_\beta A^m, \quad \mathcal{F}_{mn} = 2\partial_{[m}A_{n]},$$

being N^{mn} the Lorentz generators for the ghosts λ, given by

$$N^{nm} = \frac{\alpha' \left(\lambda\gamma^{mn}\omega\right)}{4}.$$

When necessary, the superfields will be expanded in components.The vertex operator for the closed string is given by the product of two open string operators $(\lambda^\alpha A_\alpha)\left(\bar{\lambda}^\alpha \bar{A}_\alpha\right)$. Then, we have for the amplitude [1]

$$\begin{aligned}
\mathcal{A} = \left\langle V_g V_h \int U_h \right\rangle &= g'^2_o g'_c \int dy_3 \left\langle \left[\left(\lambda^\alpha A^1_\alpha(z)\right)\left(\bar{\lambda}^\alpha \bar{A}^1_\alpha(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right) U_h(y_3)\right\rangle \\
&= g'^2_o g'_c \int dy_3 \langle \left[\left(\lambda^\alpha A^1_\alpha(z)\right)\left(\bar{\lambda}^\alpha \bar{A}^1_\alpha(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right) \\
&\quad \times \left(\partial\theta^\alpha A^3_\alpha + A^3_m \Pi^m + d_\alpha W^\alpha_3 + \frac{1}{2}N^{nm}\mathcal{F}^3_{nm}\right)\rangle.
\end{aligned} \tag{3.3}$$

A way to simplify an expression similar to this is given in ([5]). We must show here that the same expression is valid in the case of mixed string amplitudes following the same steps. First of all we must note that the first term of the integrated operator has null OPE with the other vertex operators. After this, we get

$$\begin{aligned}
\mathcal{A} = g'^2_o g'_c \int dy_3 &\langle \left[\left(\lambda^\alpha A^1_\alpha(z)\right)\left(\bar{\lambda}^\alpha \bar{A}^1_\alpha(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right) \\
&\times \left(A^3_m \Pi^m + d_\alpha W^\alpha_3 + \frac{1}{2}N^{nm}\mathcal{F}^3_{nm}\right)\rangle.
\end{aligned} \tag{3.4}$$

We also have that the OPE of any vertex operator with $A^2(y_2)$ will have null result by fixing $y_2 \rightharpoonup \infty$. For the next term, we need the standard OPE

$$: \Pi^m(y_3)\, e^{ik\cdot x}(z) :\sim ik_1^m \alpha\prime \left[\frac{1}{z-y_3}+\frac{1}{\bar{z}-y_3}\right] e^{ik\cdot x}(z)$$

to obtain

$$i\alpha' \int dy_3 k_1^m \alpha\prime \left[\frac{1}{z-y_3}+\frac{1}{\bar{z}-y_3}\right] \langle \left[\left(\lambda^\alpha A_\alpha^1(z)\right)\left(\bar{\lambda}^\alpha \bar{A}_\alpha^1(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right) A_m^3 \rangle . \qquad (3.5)$$

Now fixing

$$\mathrm{Im}(z) = ia, \quad \mathrm{Re}(z) = 0$$

the term in eq. (3.5) is also null

$$i\alpha' \int dy_3 k_1^m \alpha\prime \left[\frac{1}{ia-y_3}+\frac{1}{-ia-y_3}\right] \langle \left[\left(\lambda^\alpha A_\alpha^1(z)\right)\left(\bar{\lambda}^\alpha \bar{A}_\alpha^1(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right) A_m^3 \rangle = 0,$$

where in the above expression a contour integral gives a null result. Them only the two last terms contribute

$$\mathcal{A} = g_o'^2 g_c' \int dy_3 \left\langle \left[\left(\lambda^\alpha A_\alpha^1(z)\right)\left(\bar{\lambda}^\alpha \bar{A}_\alpha^1(\bar{z})\right)\right]\left(\lambda A^2(y_2)\right)\left(d_\alpha W_3^\alpha + \frac{1}{2} N^{nm}\mathcal{F}_{nm}^3\right)\right\rangle .$$

For the next term, we must use the OPE

$$d_\alpha(z_i)\, V(z_j) \sim -\frac{\alpha'}{2}\frac{D_\alpha V}{z_j - z_i} - \frac{\alpha'}{2}\frac{\bar{D}_\alpha V}{\bar{z}_j - z_i}$$

and we arrive in

$$\begin{aligned}\mathcal{A}_1 = & \; g_o'^2 g_c' \frac{\alpha'}{2}\int \frac{dy_3}{z-y_3}\left\langle D_\alpha\left(\lambda A^1(z)\right)\left(\overline{\lambda A}^1(\bar{z})\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle \\ & -g_o'^2 g_c' \frac{\alpha'}{2}\int \frac{dy_3}{\bar{z}-y_3}\left\langle \bar{D}_\alpha\left(\lambda A^1(z)\right)\left(\overline{\lambda A}^1(\bar{z})\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle .\end{aligned}$$

Fixing $z = ia$ in the last equation and solving the integrals we get

$$\mathcal{A}_1 = +g_o'^2 g_c' \pi i\alpha' \left\langle D_\alpha\left(\lambda A^1\right)\left(\overline{\lambda A}^1\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle + g_o'^2 g_c' \pi i\alpha' \left\langle \left(\lambda A^1\right)\bar{D}_\alpha\left(\overline{\lambda A}^1\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle .$$

After solving the OPEs, we just have zero modes and there is no difference between holomorphic and antiholomorphic terms, i.e.,

$$\mathcal{A}_1 = +g_o'^2 g_c' \pi i\alpha' \left\langle D_\alpha\left(\lambda A^1\right)\left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle + g_o'^2 g_c' \pi i\alpha' \left\langle \left(\lambda A^1\right) D_\alpha\left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) W_3^\alpha\right\rangle ,$$

where the symbol $\sim$ above of A^1 is to emphasize that the momenta are equal but the polarizations are different. Now, using the equations of motion (3.2)

$$D_\alpha(\lambda A) = -(Q)\, A_\alpha + (\lambda\gamma^m)_\alpha A_m,$$

we obtain

$$\mathcal{A}_1 = +g_o'^2 g_c' \pi i\alpha' \left\langle D_\alpha \left(\lambda A^1\right)\left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) W^\alpha \right\rangle + g_o'^2 g_c' \pi i\alpha' \left\langle \left(\lambda A^1\right) D_\alpha \left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) W_3^\alpha \right\rangle$$
$$= +g_o'^2 g_c' \pi i\alpha' \left\langle \left[-\left(Q\right) A_\alpha^1 + \left(\lambda\gamma^m\right)_\alpha A_m^1\right] \left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) W_3^\alpha \right\rangle$$
$$+g_o'^2 g_c' \pi i\alpha' \left\langle \left(\lambda A^1\right) \left[-\left(Q\right) \tilde{A}_\alpha^1 + \left(\lambda\gamma^m\right)_\alpha \tilde{A}_m^1\right] \left(\lambda A^2\right) W_3^\alpha \right\rangle.$$

Using the fact that an exact BRST term decouples, we obtain

$$\mathcal{A}_1 = +g_o'^2 g_c' \pi i\alpha' \left\langle A_m^1 \left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle$$
$$+g_o'^2 g_c' \pi i\alpha' \left\langle \tilde{A}_m^1 \left(\lambda A^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle$$
$$-g_o'^2 g_c' \pi i\alpha' \left\langle A_\alpha^1 \left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) Q W_3^\alpha \right\rangle$$
$$+g_o'^2 g_c' \pi i\alpha' \left\langle \left(\lambda A^1\right) \tilde{A}_\alpha^1 \left(\lambda A^2\right) Q W_3^\alpha \right\rangle,$$

and using

$$QW^\alpha = \frac{1}{4}\left(\lambda\gamma^{mn}\right)^\alpha \mathcal{F}_{mn}$$

we arrive in

$$\mathcal{A}_1 = +g_o'^2 g_c' \pi i\alpha' \left\langle A_m^1 \left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle$$
$$+g_o'^2 g_c' \pi i\alpha' \left\langle \tilde{A}_m^1 \left(\lambda A^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle$$
$$-g_o'^2 g_c' \frac{\pi i\alpha'}{4} \left\langle \left(\lambda\gamma^{mn} A^1\right)\left(\lambda \tilde{A}^1\right)\left(\lambda A^2\right) \mathcal{F}_{mn}^3 \right\rangle$$
$$+g_o'^2 g_c' \frac{\pi i\alpha'}{4} \left\langle \left(\lambda A^1\right)\left(\lambda\gamma^{mn} \tilde{A}^1\right)\left(\lambda A^2\right) \mathcal{F}_{mn}^3 \right\rangle.$$

For the last term of the expression (3.4), we have

$$\mathcal{A}_2 = g_o'^2 g_c' \int dy_3 \left\langle \left(\lambda A^1\right)\left(\bar{\lambda}\bar{A}^1\right)\left(\lambda A^2\right) \frac{1}{2} N^{mn} F_{mn}^3 \right\rangle \tag{3.6}$$
$$= g_o'^2 g_c' \int dy_3 \frac{\alpha'}{8\left(z-y_3\right)} \left\langle \left(\lambda\gamma^{mn} A^1\right)\left(\bar{\lambda}\bar{A}^1\right)\left(\lambda A^2\right) F_{mn}^3 \right\rangle$$
$$+g_o'^2 g_c' \int dy_3 \frac{\alpha'}{8\left(\bar{z}-y_3\right)} \left\langle \left(\lambda A^1\right)\left(\bar{\lambda}\gamma^{mn}\bar{A}^1\right)\left(\lambda A^2\right) F_{mn}^3 \right\rangle$$
$$+g_o'^2 g_c' \int dy_3 \frac{\alpha'}{8\left(y_2-y_3\right)} \left\langle \left(\lambda A^1\right)\left(\bar{\lambda}\bar{A}^1\right)\left(\lambda\gamma^{mn} A^2\right) F_{mn}^3 \right\rangle,$$

where we have used the OPE

$$N^{mn}\left(y_3\right)\lambda^\alpha\left(z\right) = \frac{\alpha'}{4\left(z-y_3\right)}\left(\lambda\gamma^{mn}\right)^\alpha.$$

Fixing above $y_2 = \infty$ and $z = ia$, we get

$$\mathcal{A}_2 = g_o'^2 g_c' \frac{\pi i\alpha'}{4} \left\langle \left(\lambda\gamma^{mn} A^1\right)\left(\bar{\lambda}\bar{A}^1\right)\left(\lambda A^2\right) F_{mn}^3 \right\rangle \tag{3.7}$$
$$-g_o'^2 g_c' \frac{\pi i\alpha'}{4} \left\langle \left(\lambda A^1\right)\left(\bar{\lambda}\gamma^{mn}\bar{A}^1\right)\left(\lambda A^2\right) F_{mn}^3 \right\rangle.$$

Adding the results (3.6) and (3.7), we finally obtain

$$\begin{aligned}\mathcal{A} &= g_o'^2 g_c' \pi i \alpha' \left\langle A_m \left(\lambda \tilde{A}^1\right) \left(\lambda A^2\right) \left(\lambda \gamma^m W^3\right) \right\rangle + g_o'^2 g_c' \pi i \alpha' \left\langle \tilde{A}_m \left(\lambda A^1\right) \left(\lambda A^2\right) \left(\lambda \gamma^m W^3\right) \right\rangle \\ &= g_o'^2 g_c' \pi i \alpha' \left\langle \left[A_m^1 \left(\lambda \tilde{A}^1\right) + \tilde{A}_m^1 \left(\lambda A^1\right)\right] \left(\lambda A^2\right) \left(\lambda \gamma^m W^3\right) \right\rangle . \end{aligned} \tag{3.8}$$

Although the starting expression (3.3) has gauge invariance, we left the proof to appendix B. At this point, we must expand the superfields in components and use the measure

$$\langle (\lambda \gamma^a \theta) \left(\lambda \gamma^b \theta\right) (\lambda \gamma^c \theta) (\theta \gamma_{abc} \theta) = 1 \tag{3.9}$$

in order to find the contribution of each component. The superfield expansion is given by

$$\begin{aligned}\lambda A &= \frac{1}{2} a_f \left(\lambda \gamma^f \theta\right) - \frac{1}{3} (\xi \gamma_m \theta) (\lambda \gamma^m \theta) - \frac{1}{32} F_{mn} (\lambda \gamma_p \theta) (\theta \gamma^{mnp} \theta) \\ &\quad + \frac{1}{60} (\lambda \gamma_m \theta)_\alpha (\theta \gamma^{mnp} \theta) (\partial_n \xi \gamma_p \theta) \dots \\ A_m &= a_m - (\xi \gamma_m \theta) - \frac{1}{8} (\theta \gamma_m \gamma^{pq} \theta) F_{pq} + \frac{1}{12} (\theta \gamma_m \gamma^{pq} \theta) (\partial_p \xi \gamma_q \theta) \dots \\ \lambda \gamma^s W &= \lambda \gamma^s \xi - \frac{1}{4} (\lambda \gamma^s \gamma^{mn} \theta) F_{mn} + \frac{1}{4} (\lambda \gamma^s \gamma^{mn} \theta)\, \partial_m \xi \gamma_n \theta + \frac{1}{48} (\lambda \gamma^s \gamma^{mn} \theta)(\theta \gamma_n \gamma^{pq} \theta)\, \partial_m F_{pq} \dots \end{aligned} \tag{3.10}$$

These expressions will be used in computations in the next section.

4. Correlation functions in components

4.1 One graviton/dilaton and two photons

As explained in the previous section, the vertex operator for closed strings can be written as the product of the vertex operators of open strings. First of all, we need to identify the NS-NS contribution in this product. From our final expression (3.8), the closed string contribution is given by

$$\left[A_m^1 \left(\lambda \tilde{A}^1\right) + \tilde{A}_m^1 \left(\lambda A^1\right)\right].$$

Using the superfield expansion, we have the following result for the NS-NS contribution

$$\begin{aligned}&\left(h_{g_1} - \frac{1}{4} \partial_{m_1} h_{h_1} \eta_{g_1 t_2} \left(\theta \gamma^{t_2} \gamma^{m_1 h_1} \theta\right)\right) \times \\ &\left(\frac{1}{2} \tilde{h}_{g_2} (\lambda \gamma^{g_2} \theta) - \frac{1}{16} \partial_{m_1} \tilde{h}_{g_2} \eta_{t_1 t_2} \left(\lambda \gamma^{t_1} \theta\right) \left(\theta \gamma^{m_1 g_2 t_2} \theta\right)\right) \\ &+ \left(\tilde{h}_{g_1} - \frac{1}{4} \partial_{m_1} \tilde{h}_{h_1} \eta_{g_1 t_2} \left(\theta \gamma^{t_2} \gamma^{m_1 h_1} \theta\right)\right) \times \\ &\left(\frac{1}{2} h_{g_2} (\lambda \gamma^{g_2} \theta) - \frac{1}{16} \partial_{m_1} h_{g_2} \eta_{t_1 t_2} \left(\lambda \gamma^{t_1} \theta\right) \left(\theta \gamma^{m_1 g_2 t_2} \theta\right)\right) \\ &= \left[\frac{1}{2} \left(h_{g_1} \tilde{h}_{g_2} + \tilde{h}_{g_1} h_{g_2}\right) (\lambda \gamma^{g_2} \theta) - \frac{1}{16} \left(h_{g_1} \partial_{m_1} \tilde{h}_{g_2} + \tilde{h}_{g_1} \partial_{m_1} h_{g_2}\right) \eta_{t_1 t_2} \left(\lambda \gamma^{t_1} \theta\right) \left(\theta \gamma^{m_1 g_2 t_2} \theta\right) \right. \\ &\quad \left. - \frac{1}{8} \left(\tilde{h}_{g_2} \partial_{m_1} h_{h_1} + h_{g_2} \partial_{m_1} \tilde{h}_{h_1}\right) \eta_{g_1 t_2} \left(\theta \gamma^{t_2} \gamma^{m_1 h_1} \theta\right) (\lambda \gamma^{g_2} \theta)\right]. \end{aligned} \tag{4.1}$$

We must be careful here to identify the NS-NS field, for when we write the closed string as the product of two open strings, each part must carry half of the momentum. For example

$$\left(h_{g_1}\partial_{m_1}\tilde{h}_{g_2}+\tilde{h}_{g_1}\partial_{m_1}h_{g_2}\right)=\left(i\frac{k^1_{m_1}}{2}h_{g_1}\tilde{h}_{g_2}+i\frac{k^1_{m_1}}{2}\tilde{h}_{g_1}h_{g_2}\right)$$
$$=\frac{1}{2}\partial_{m_1}\left(h_{g_1}\tilde{h}_{g_2}+\tilde{h}_{g_1}h_{g_2}\right).$$

From this, we can see that, like in Ramond-Neveu-Schwarz, only the symmetric part of the NS-NS sector contributes. Its traceless part is identified with the graviton. We will see later how the two form will come from the RR sector. Now, making the identification

$$h_{g_1}\tilde{h}_{g_2}+\tilde{h}_{g_1}h_{g_2}=2h_{g_1g_2}$$

we obtain the graviton contribution

$$\Big[h_{g_1g_2}\left(\lambda\gamma^{g_2}\theta\right)-\frac{1}{16}\partial_{m_1}h_{g_2g_1}\eta_{t_1t_2}\left(\lambda\gamma^{t_1}\theta\right)\left(\theta\gamma^{m_1g_2t_2}\theta\right)$$
$$-\frac{1}{8}\partial_{m_1}h_{h_1g_2}\eta_{g_1t_2}\left(\theta\gamma^{t_2}\gamma^{m_1h_1}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\Big]. \quad (4.2)$$

After the identification of the NS-NS contribution, we must go back to the expression (3.8) and consider only the photon contribution from (3.10) to obtain

$$\mathcal{A}=g_o'^2g_c'\pi i\alpha'\langle\Big[h_{g_1g_2}\left(\lambda\gamma^{g_2}\theta\right)-\frac{1}{16}\partial_{m_1}h_{g_2g_1}\eta_{t_1t_2}\left(\lambda\gamma^{t_1}\theta\right)\left(\theta\gamma^{m_1g_2t_2}\theta\right)$$
$$-\frac{1}{8}\partial_{m_1}h_{h_1g_2}\eta_{g_1t_2}\left(\theta\gamma^{t_2}\gamma^{m_1h_1}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\Big]$$
$$\times\left(\frac{1}{2}a^2_{f_2}\left(\lambda\gamma^{f_2}\theta\right)-\frac{1}{32}F^2_{m_2f_2}\left(\lambda\gamma_p\theta\right)\left(\theta\gamma^{m_2f_2p}\theta\right)\right)$$
$$\times\left(-\frac{1}{4}\left(\lambda\gamma^{g_1}\gamma^{m_3f_3}\theta\right)F^3_{m_3f_3}+\frac{1}{48}\left(\lambda\gamma^{g_1}\gamma^{m_3n}\theta\right)\left(\theta\gamma_n\gamma^{n_3f_3}\theta\right)\partial_{m_3}F^3_{n_3f_3}\right)\rangle.$$

As we know from eq. (3.9), only terms with five thetas contribute to the amplitude. Then, we have

$$\mathcal{A}=g_o'^2g_c'\frac{\pi i\alpha'}{96}\langle h_{g_1g_2}a^2_{f_2}\partial_{m_3}F^3_{n_3f_3}\left(\lambda\gamma^{g_1}\gamma^{m_3n}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma_n\gamma^{n_3f_3}\theta\right)\rangle$$
$$+g_o'^2g_c'\frac{\pi i\alpha'}{128}\langle h_{g_1g_2}F^2_{m_2f_2}F^3_{m_3f_3}\left(\lambda\gamma^{g_1}\gamma^{m_3f_3}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma_p\theta\right)\left(\theta\gamma^{m_2f_2p}\theta\right)\rangle$$
$$+g_o'^2g_c'\frac{\pi i\alpha'}{128}\langle a^2_{f_2}\partial_{m_1}\tilde{h}_{g_2g_1}F^3_{m_3f_3}\eta_{t_1t_2}\left(\lambda\gamma^{g_1}\gamma^{m_3f_3}\theta\right)\left(\lambda\gamma^{t_1}\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma^{m_1g_2t_2}\theta\right)\rangle$$
$$+g_o'^2g_c'\frac{\pi i\alpha'}{64}\langle a^2_{f_2}\partial_{m_1}h_{g_1g_2}\eta_{t_1t_2}F^3_{m_3f_3}\left(\lambda\gamma^{t_1}\gamma^{m_3f_3}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma^{t_2}\gamma^{m_1g_1}\theta\right)\rangle.$$

In order to solve the above expression, we need to use the identity (A.4) and successive times the identities (A.1) and (A.2) described in appendix A. Solving term by term with the help of the GAMMA package [8] we obtain

$$\begin{aligned}\mathcal{A}_1 &= \pi i\alpha'\Big(-\frac{1}{17280}h^{g_1g_2}a_2^{f_2}\partial^{g_2}F^3_{f_2g_1} - \frac{7}{34560}h_{g_1g_2}a^2_{f_2}\partial^{g_1}F^3_{f_2g_2}\Big)\\ &= -\frac{\pi i\alpha'}{3840}h^{g_1g_2}a_2^{f_2}\partial^{g_2}F^3_{f_2g_1} = \frac{\pi i\alpha'}{3840}h^{g_1g_2}\partial^{g_2}a_2^{f_2}F^3_{f_2g_1}.\end{aligned}$$

In a similar way, we can obtain the second term

$$\mathcal{A}_2 = -\frac{\pi i\alpha'}{2304}h^{g_2}_{g_1}F^2_{f_3g_2}F_3^{f_3g_1} - \frac{\pi i\alpha'}{23040}h^{g_1}_{g_1}F^2_{f_3g_1}F_3^{f_3g_1}.$$

The third one is given by

$$\begin{aligned}\mathcal{A}_3 &= -\frac{\pi i\alpha'}{7680}a^2_{f_2}\partial^{m_1}h^{f_2g_1}F^3_{g_1m_1} + \frac{\pi i\alpha'}{23040}\partial^{m_1}h^{g_1}_{g_1}a_2^{g_1}F^3_{g_1m_1}\\ &= \frac{\pi i\alpha'}{7680}\partial^{m_1}a^2_{f_2}h^{f_2g_1}F^3_{g_1m_1} - \frac{\pi i\alpha'}{23040}h^{g_1}_{g_1}\partial^{m_1}a_2^{g_1}F^3_{g_1m_1}\\ &= \frac{\pi i\alpha'}{7680}h^{f_2g_1}\partial^{m_1}a^2_{f_2}F^3_{g_1m_1} + \frac{\pi i\alpha'}{46080}h^{g_1}_{g_1}F_2^{g_1m_1}F^3_{g_1m_1},\end{aligned}$$

and the fourth is

$$\begin{aligned}\mathcal{A}_4 &= -g_o'^2g_c'\frac{1}{7680}a^2_{f_2}\partial^{m_1}h^{f_2g_2}F^3_{g_2m_1}\\ &= g_o'^2g_c'\frac{1}{7680}h^{f_2g_2}\partial^{m_1}a^2_{f_2}F^3_{g_2m_1}.\end{aligned}$$

Adding all terms, we obtain

$$\begin{aligned}\mathcal{A} &= g_o'^2g_c'\pi i\alpha'\Big(\frac{1}{3840}h^{g_1g_2}\partial^{g_2}a_2^{f_2}F^3_{f_2g_1} - \frac{1}{2304}h^{g_2}_{g_1}F^2_{f_3g_2}F_3^{f_3g_1}\\ &\qquad +\frac{1}{7680}h^{f_2g_1}\partial^{m_1}a^2_{f_2}F^3_{g_1m_1} + \frac{1}{7680}h^{f_2g_2}\partial^{m_1}a^2_{f_2}F^3_{g_2m_1}\Big) +\\ &\quad -\frac{\pi i\alpha'}{11520}h^{g_1}_{g_1}F^2_{f_2m_1}F_3^{f_2m_1}\\ &= g_o'^2g_c'\pi i\alpha'\Big(\frac{1}{3840}h^{g_1g_2}F_2^{g_2f_2}F^3_{f_2g_1} - \frac{1}{2304}h^{g_2}_{g_1}F^2_{f_3g_2}F_3^{f_3g_1}\Big)\\ &= -g_o'^2g_c'\frac{\pi i\alpha'}{1440}h^{g_1}_{g_2}F_2^{g_2f_2}F^3_{f_2g_1} - \frac{\pi i\alpha'}{11520}h^{g_1}_{g_1}F^2_{f_2m_1}F_3^{f_2m_1}.\end{aligned}$$

In the gauge $k^mh_{mn} = 0$, the relation $h^{g_1}_{g_1} = 4\Phi$ is valid [6], and we finally get

$$\mathcal{A} = \frac{\pi i\alpha'}{720}g_o'^2g_c'\left(-\frac{\pi i\alpha'}{2}h^{g_1}_{g_2}F_2^{g_2f_2}F^3_{f_2g_1} - \frac{\pi i\alpha'}{4}\Phi F^2_{f_2m_1}F_3^{f_2m_1}\right). \tag{4.3}$$

Comparing this last expression with (2.1), we see that up to a overall factor we have the right result.

4.2 One graviton/dilaton and two photinos

From the pure spinor viewpoint, we need the graviton and the photino contribution to the vertex operators which are respectively given by (4.2) and (3.10). We then obtain

$$\begin{aligned}\mathcal{A} = g_o'^2 g_c' \pi i\alpha' \langle \Big[& h_{g_1g_2} \left(\lambda\gamma^{g_2}\theta\right) - \frac{1}{16}\partial_{m_1} h_{g_2g_1}\eta_{t_1t_2} \left(\lambda\gamma^{t_1}\theta\right)\left(\theta\gamma^{m_1g_2t_2}\theta\right) \\ & -\frac{1}{8}\partial_{m_1} h_{h_1g_2}\eta_{g_1t_2}\left(\theta\gamma^{t_2}\gamma^{m_1h_1}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\Big] \\ & \times\left(-\frac{1}{3}\left(\xi^2\gamma_r\theta\right)\left(\lambda\gamma^r\theta\right)+\frac{1}{60}\left(\lambda\gamma_r\theta\right)\left(\theta\gamma^{rst}\theta\right)\left(\partial_s\xi^2\gamma_t\theta\right)\right) \\ & \times\left(\lambda\gamma^m\xi^3+\frac{1}{4}\left(\lambda\gamma^m\gamma^{pq}\theta\right)\partial_p\xi^3\gamma_q\theta\right)\rangle. \end{aligned} \tag{4.4}$$

In the expression above, we have four terms with five thetas given by

$$\begin{aligned}\mathcal{A} = 2g_o'^2 g_c'\pi i\alpha'\langle\ & \frac{1}{24}h_{mg2}\left(\lambda\gamma^m\gamma^{m_3f_3}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma_{f_2}\xi^2\right)\partial_{m_3}\xi^3\gamma_{f_3}\theta\rangle \\ & +2\pi i\alpha'\langle\frac{1}{120}h_{mg2}\left(\lambda\gamma^m\xi^3\right)\left(\partial_{m_2}\xi^2\gamma_{f_2}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma_r\theta\right)\left(\theta\gamma^{rm_2f_2}\theta\right)\rangle \\ & +2\pi i\alpha'\langle\frac{1}{96}\partial_{m_1}h_{g_1g_2}\left(\lambda\gamma^{g_1}\xi^3\right)\left(\xi^2\gamma_{f_2}\theta\right)\left(\lambda\gamma_p\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma^{m_1g_2p}\theta\right)\rangle \\ & 2\pi i\alpha'\langle\frac{1}{48}\partial_{m_1}h_{g_1g_2}\left(\lambda\gamma_m\xi^3\right)\left(\xi^2\gamma_{f_2}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma^{f_2}\theta\right)\left(\theta\gamma^{mm_1g_1}\theta\right)\rangle. \end{aligned} \tag{4.5}$$

Using now the identity

$$\xi_2^\alpha\xi_3^\beta = \frac{1}{16}\left(\gamma_a\right)^{\alpha\beta}\left(f^a\right)+\frac{1}{96}\left(\gamma_{abc}\right)^{\alpha\beta}\left(f^{abc}\right)+\frac{1}{3840}\left(\gamma_{abcde}\right)^{\alpha\beta}\left(f^{abcde}\right), \tag{4.6}$$

where

$$f^{a\dots}=\xi^1\gamma^{a\dots}\xi^2,$$

we have

$$\xi_2^\alpha\partial_{m_3}\xi_3^\beta = \frac{1}{16}\left(\gamma_a\right)^{\alpha\beta}\left(f^a_{1m_3}\right)+\frac{1}{96}\left(\gamma_{abc}\right)^{\alpha\beta}\left(f^{abc}_{1m_3}\right)+\frac{1}{3840}\left(\gamma_{abcde}\right)^{\alpha\beta}\left(f^{abcde}_{1m_3}\right).$$

Therefore the first term gives

$$\begin{aligned}\mathcal{A}_1 = g_o'^2 g_c' 2\pi i\alpha'\langle & \frac{1}{24}h_{g_1g_2}\left(\lambda\gamma^{g_1}\gamma^{m_3f_3}\theta\right)\left(\lambda\gamma^{g_2}\theta\right)\left(\lambda\gamma^{f_2}\theta\right) \\ & \times\left(\frac{1}{16}(\theta\gamma_{f_2}\gamma_a\gamma_{f_3}\theta)\left(f^a_{1m_3}\right)+\frac{1}{96}(\theta\gamma_{f_2}\gamma_{abc}\gamma_{f_3}\theta)\left(f^{abc}_{1m_3}\right)+\frac{1}{3840}(\theta\gamma_{f_2}\gamma_{abcde}\gamma_{f_3}\theta)\left(f^{abcde}_{1m_3}\right)\right)\rangle. \end{aligned}$$

Using now the identities (A.5), (A.6), (A.7) and successive times the identities (A.1), (A.2) and (A.3), we obtain that only the first term contributes

$$\begin{aligned}\mathcal{A}_1 &= g_o'^2 g_c' 2\pi i\alpha'\left[-\frac{1}{17280}h_{g_1g_2}\xi^2\gamma^{g_1}\partial^{g_2}\xi^3-\frac{1}{4320}h_{g_1g_2}\xi_2\gamma^{g_2}\partial^{g_1}\xi_3\right] \\ &= -g_o'^2 g_c'\frac{2\pi i\alpha'}{3456}h_{g_1g_2}\xi_2\gamma^{g_1}\partial^{g_2}\xi_3. \end{aligned}$$

The next term in (4.5) is

$$\mathcal{A}_2 = g_o'^2 g_c' \frac{2\pi i\alpha'}{120} \langle h_{g_1g_2} \left(\lambda\gamma^{g_1}\xi^3\right) \left(\partial_{m_2}\xi^2\gamma_{f_2}\theta\right) \left(\lambda\gamma^{g_2}\theta\right) \left(\lambda\gamma_r\theta\right) \left(\theta\gamma^{rm_2f_2}\theta\right)\rangle.$$

Following the same steps described above, we obtain for the second term

$$\mathcal{A}_2 = -g_o'^2 g_c' \frac{2\pi i\alpha'}{17280} h_{g_1g_2}\xi_3\gamma^{g_1}\partial^{g_2}\xi_2.$$

For the third term we get

$$\mathcal{A}_3 = g_o'^2 g_c' \frac{2\pi i\alpha'}{96} \langle \partial_{m_1} h_{g_1g_2} \left(\lambda\gamma^{g_1}\xi^3\right) \left(\xi^2\gamma_{f_2}\theta\right) \left(\lambda\gamma_p\theta\right) \left(\lambda\gamma^{f_2}\theta\right) \left(\theta\gamma^{m_1g_2p}\theta\right)\rangle,$$

which has null result. In fact, there is no way to contract a kinetic term for the graviton with two photinos giving a non null result and

$$\mathcal{A}_3 = 0.$$

For the last term we have

$$\mathcal{A}_4 = g_o'^2 g_c' 2\pi i\alpha' \langle \frac{1}{48}\partial_{m_1} h_{g_1g_2} \left(\lambda\gamma_m\xi^3\right) \left(\xi^2\gamma_{f_2}\theta\right) \left(\lambda\gamma^{g_2}\theta\right) \left(\lambda\gamma^{f_2}\theta\right) \left(\theta\gamma^{mm_1g_1}\theta\right)\rangle,$$

which is also null for the same reason as before. Finally, adding all terms we obtain

$$\begin{aligned}\mathcal{A} &= -g_o'^2 g_c' \frac{\pi i\alpha'}{1440} h_{g_1g_2}\xi_2\gamma^{g_1}\partial^{g_2}\xi_3 \\ &= g_o'^2 g_c' \frac{\pi i\alpha'}{720}\left(-\frac{1}{2} h_{g_1g_2}\xi_2\gamma^{g_1}\partial^{g_2}\xi_3\right).\end{aligned}$$

In this case the correlation of one dilaton and of two photinos gives a null result using the photino's equation of motion. The amplitude above is proportional to the respective term in the effective action (2.1), with the same overall factor of the eq. (4.3).

4.3 One gravitino/dilatino, one photon and one photino

In the pure spinor computation, we need the gravitino contribution for the vertex operator. This is given by

$$\begin{aligned}A^1_\alpha \tilde{A}^1_m &= -\frac{1}{2} h_{g_1}\left(\gamma^{g_1}\theta\right)_\alpha \left(\tilde{\xi}\gamma_m\theta\right) + \frac{1}{24} h_{g_1}\left(\gamma^{g_1}\theta\right)_\alpha \left(\theta\gamma_m\gamma^{pq}\theta\right)\left(\partial_p\tilde{\xi}\gamma_q\theta\right) \\ &\quad -\frac{1}{3}\tilde{h}_m\left(\xi\gamma_r\theta\right)\left(\gamma^r\theta\right)_\alpha + \frac{1}{12}\left(\xi\gamma_r\theta\right)\left(\gamma^r\theta\right)_\alpha\left(\theta\gamma_m\gamma^{pq}\theta\right)\partial_p\tilde{h}_q \\ &\quad +\frac{1}{16}\partial_r h_s\left(\gamma_t\theta\right)_\alpha\left(\theta\gamma^{rst}\theta\right)\left(\tilde{\xi}\gamma_m\theta\right) + \frac{1}{60}\tilde{h}_m\left(\gamma_r\theta\right)_\alpha\left(\theta\gamma^{rst}\theta\right)\left(\partial_s\xi\gamma_t\theta\right).\end{aligned}$$

Then we have

$$\begin{aligned}\tilde{A}^1_\alpha A^1_m + A^1_\alpha \tilde{A}^1_m &= -\frac{1}{2}\left(\gamma^{g_1}\theta\right)_\alpha\left[\left(h_{g_1}\tilde{\xi} + \tilde{h}_{g_1}\xi\right)\gamma_m\theta\right] \\ &\quad +\frac{1}{24}\left(\gamma^{g_1}\theta\right)_\alpha\left(\theta\gamma_m\gamma^{pq}\theta\right)\left[\left(h_{g_1}\partial_p\tilde{\xi} + \tilde{h}_{g_1}\partial_p\xi\right)\gamma_q\theta\right]\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{3}\left[\left(h_m\tilde{\xi}+\tilde{h}_m\xi\right)\gamma_r\theta\right](\gamma^r\theta)_\alpha \\
&+\frac{1}{12}\left[\left(\partial_p\tilde{h}_q\xi+\partial_p h_q\tilde{\xi}\right)\gamma_r\theta\right](\gamma^r\theta)_\alpha(\theta\gamma_m\gamma^{pq}\theta) \\
&+\frac{1}{16}(\gamma_t\theta)_\alpha\left(\theta\gamma^{rst}\theta\right)\left[\left(\partial_r h_s\tilde{\xi}+\partial_r\tilde{h}_s\xi\right)\gamma_m\theta\right] \\
&+\frac{1}{60}(\gamma_r\theta)_\alpha\left(\theta\gamma^{rst}\theta\right)\left[\left(\tilde{h}_m\partial_s\xi+h_m\partial_s\tilde{\xi}\right)\gamma_t\theta\right].
\end{aligned}
$$

Using the identification

$$
h_m\tilde{\xi}+\tilde{h}_m\xi=2\psi_m,\left(h_{g_1}\partial_p\tilde{\xi}+\tilde{h}_{g_1}\partial_p\xi\right)=\partial_p\psi_{g_1}
$$

and being careful with the terms with derivatives, we obtain

$$
\begin{aligned}
\lambda\tilde{A}^1A^1_m+\lambda A^1\tilde{A}^1_m=&-(\lambda\gamma^{g_1}\theta)(\psi_{g_1}\gamma_m\theta)+\frac{1}{24}(\lambda\gamma^{g_1}\theta)(\theta\gamma_m\gamma^{pq}\theta)(\partial_p\psi_{g_1}\gamma_q\theta) \\
&-\frac{2}{3}(\psi_m\gamma_r\theta)(\lambda\gamma^r\theta)+\frac{1}{12}(\partial_p\psi_q\gamma_r\theta)(\lambda\gamma^r\theta)(\theta\gamma_m\gamma^{pq}\theta) \\
&+\frac{1}{16}(\lambda\gamma_t\theta)\left(\theta\gamma^{rst}\theta\right)(\partial_r\psi_s\gamma_m\theta)+\frac{1}{60}(\lambda\gamma_r\theta)\left(\theta\gamma^{rst}\theta\right)(\partial_m\psi_s\gamma_t\theta).
\end{aligned}
\tag{4.7}
$$

Now, we go back to the general expression (3.8) and consider the contribution of the photon to one of the open strings and the photino to the other. We obtain

$$
\begin{aligned}
\mathcal{A}=&\,g_o'^2g_c'\pi i\alpha'\left\langle\left(\lambda\tilde{A}A_m+\lambda A\tilde{A}_m\right)(\lambda A^2)(\lambda\gamma^m W)\right\rangle= \\
=&\,g_o'^2g_c'\pi i\alpha'\langle\Big[-(\lambda\gamma^{g_1}\theta)(\psi_{g_1}\gamma_m\theta)-\frac{2}{3}(\psi_m\gamma_r\theta)(\lambda\gamma^r\theta) \\
&+\frac{1}{24}(\lambda\gamma^{g_1}\theta)(\theta\gamma_m\gamma^{m_1q}\theta)(\partial_{m_1}\psi_{g_1}\gamma_q\theta)+\frac{1}{12}(\partial_{m_1}\psi_{g_1}\gamma_r\theta)(\lambda\gamma^r\theta)(\theta\gamma_m\gamma^{m_1g_1}\theta) \\
&+\frac{1}{16}(\lambda\gamma_t\theta)\left(\theta\gamma^{m_1g_1t}\theta\right)(\partial_{m_1}\psi_{g_1}\gamma_m\theta)+\frac{1}{60}(\lambda\gamma_r\theta)\left(\theta\gamma^{rg1t}\theta\right)(\partial_m\psi_{g_1}\gamma_t\theta)\Big] \\
&\times\left(\frac{1}{2}a^2_{f_2}\left(\lambda\gamma^{f_2}\theta\right)-\frac{1}{3}(\xi^2\gamma_t\theta)(\lambda\gamma^t\theta)-\frac{1}{32}F^2_{m_2f_2}(\lambda\gamma_p\theta)\left(\theta\gamma^{m_2f_2p}\theta\right)\right) \\
&\times\Bigg(\lambda\gamma_m\xi^3-\frac{1}{4}\left(\lambda\gamma_m\gamma^{m_3f_3}\theta\right)F^3_{m_3f_3}+\frac{1}{4}(\lambda\gamma_m\gamma^{m_3s}\theta)\partial_{m_3}\xi^3\gamma_s\theta \\
&+\frac{1}{48}(\lambda\gamma_m\gamma^{rs}\theta)\left(\theta\gamma_s\gamma^{m_3f_3}\theta\right)\partial_rF^3_{m_3f_3}\Bigg)\rangle.
\end{aligned}
$$

There will be ten terms with five thetas and, as we have two fermions, we also need to expand them using the identity (4.6) to obtain a total of thirty terms. The details are described in appendix D, and the result is given by

$$
\mathcal{A}=-g_o'^2g_c'\frac{\pi i\alpha'}{720}(F^2_{g_1f_2}\xi_3\gamma^{f_2}\psi^{g_1}+F^3_{m_3f_3}\xi_2\gamma^{f_3}\psi^{m_3}).
$$

Again, we get that this is proportional to the respective term of the effective action (2.1) with the right overall factor. Note that, as in all other terms, this amplitude is symmetric by the exchange of the two open strings. The dilatino-photino-photon correlation can be found by this shortcut. We must take the photon contribution from the fixed operator and the photino from the integrated one. Using the fact that the amplitude is symmetric by this exchange, we obviously obtain the right result. We then have

$$\mathcal{A} = -g_o'^2 g_c' \frac{\pi i \alpha'}{720} (\frac{1}{4} F^3_{g_1 f_2} \xi_2 \gamma^{g_1 f_2} \lambda),$$

which agree with the desired result.

4.4 Kalb-Ramond and two photons

In the type I superstring, the two form does not come from the NS-NS sector, as shown before. In fact the two form comes from the RR sector and only appears as a field strength. The RR contribution to the closed string vertex operator comes from

$$\begin{aligned}\lambda A^1 \tilde{A}^1_m + \lambda \tilde{A}^1 A^1_m = & \left(-\frac{1}{3}(\xi\gamma_n\theta)(\lambda\gamma^n\theta) + \frac{1}{60}(\lambda\gamma_m\theta)_\alpha (\theta\gamma^{mnp}\theta)(\partial_n\xi\gamma_p\theta)\right) \\ & \times \left(-\left(\tilde{\xi}\gamma_m\theta\right) + \frac{1}{12}(\theta\gamma_m\gamma^{pq}\theta)\left(\partial_p\tilde{\xi}\gamma_q\theta\right)\right) \\ & + \left(-\frac{1}{3}\left(\tilde{\xi}\gamma_n\theta\right)(\lambda\gamma^n\theta) + \frac{1}{60}(\lambda\gamma_m\theta)_\alpha (\theta\gamma^{mnp}\theta)\left(\partial_n\tilde{\xi}\gamma_p\theta\right)\right) \\ & \times \left(-(\xi\gamma_m\theta) + \frac{1}{12}(\theta\gamma_m\gamma^{pq}\theta)(\partial_p\xi\gamma_q\theta)\right).\end{aligned}$$

Making the identification

$$\tilde{\xi}^a \xi^\beta + \xi^\alpha \tilde{\xi}^\beta = 2F^{\alpha\beta},$$

we have only one contribution given by

$$\lambda A^1 \tilde{A}^1_m + \lambda \tilde{A}^1 A^1_m = -\frac{2}{3}(\lambda\gamma^n\theta)(\theta\gamma_m)_\alpha F^{\alpha\beta}(\gamma_n\theta)_\beta. \tag{4.8}$$

The other terms have five thetas and do not contribute to the amplitude. We have then

$$\begin{aligned}\mathcal{A} = & g_o'^2 g_c' \pi i \alpha' \langle \left(-\frac{2}{3}(\lambda\gamma^n\theta)(\theta\gamma_m)_\alpha F^{\alpha\beta}(\gamma_n\theta)_\beta\right) \\ & \times \left(\frac{1}{2} a^2_{f_2}\left(\lambda\gamma^{f_2}\theta\right) - \frac{1}{32} F^2_{m_2 f_2}(\lambda\gamma_p\theta)\left(\theta\gamma^{m_2 f_2 p}\theta\right)\right) \\ & \times \left(-\frac{1}{4}\left(\lambda\gamma^{g_1}\gamma^{m_3 f_3}\theta\right) F^3_{m_3 f_3} + \frac{1}{48}(\lambda\gamma^{g_1}\gamma^{m_3 n}\theta)\left(\theta\gamma_n\gamma^{n_3 f_3}\theta\right)\partial_{m_3} F^3_{n_3 f_3}\right)\rangle.\end{aligned} \tag{4.9}$$

We see that there is just one contribution given by

$$\mathcal{A} = g_o'^2 g_c' \frac{\pi i \alpha'}{12} \langle (\lambda\gamma^n\theta)(\theta\gamma_m)_\alpha F^{\alpha\beta}(\gamma_n\theta)_\beta a^2_{f_2}\left(\lambda\gamma^{f_2}\theta\right)(\lambda\gamma^m\gamma^{pq}\theta) F^3_{pq}\rangle.$$

Using now the identity (4.6), the RR field can be expanded

$$F^{\alpha\beta} = \gamma_a^{\alpha\beta} F^a + \frac{1}{96}\gamma^{\alpha\beta}_{abc} H^{abc} + \frac{1}{3840}\gamma^{\alpha\beta}_{abcde} F^{abcde}.$$

In the type I superstring the term that survives is the three-form, and we obtain

$$\mathcal{A} = g_o'^2 g_c' \frac{\pi i \alpha'}{12} \langle H^{abc} F^3_{pq} a^2_{f_2} (\lambda\gamma^m\gamma^{pq}\theta)(\lambda\gamma^n\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma_m\gamma_{abc}\gamma_n\theta)\rangle.$$

In order to solve this term we must use the identities (A.4), (A.6) and successive applications of the identities (A.1), (A.2) and (A.3). The result is

$$\mathcal{A} = g_o'^2 g_c' \frac{\pi i \alpha'}{720}\left(\frac{1}{8} a^2_{f_2} F^3_{m_3 f_3} H^{f_2 f_3 m_3}\right),$$

and it is proportional to the expression (2.1), as desired. This term is very important because it gives origin to a coupling which will cancel the mixed anomaly of SO(32) type I superstring.

4.5 Kalb-Ramond and two photinos

The RR contribution to the closed string is given by (4.8), and we have the amplitude

$$\begin{aligned}\mathcal{A} = g_o'^2 g_c' \pi i \alpha' \langle &\left(-\frac{2}{3}(\lambda\gamma^n\theta)(\theta\gamma_m)_\alpha F^{\alpha\beta}(\gamma_n\theta)_\beta\right) \\ \times &\left(-\frac{1}{3}(\xi^2\gamma_r\theta)(\lambda\gamma^r\theta) + \frac{1}{60}(\lambda\gamma_r\theta)(\theta\gamma^{rst}\theta)(\partial_s\xi^2\gamma_t\theta)\right) \\ \times &\left(\lambda\gamma^m\xi^3 + \frac{1}{4}(\lambda\gamma^m\gamma^{pq}\theta)\,\partial_p\xi^3\gamma_q\theta\right)\rangle.\end{aligned}$$

The unique term which has five thetas in the last equation is the following

$$\begin{aligned}\mathcal{A} &= g_o'^2 g_c' \frac{2\pi i\alpha'}{9} \langle(\lambda\gamma^m\xi^3)(\xi^2\gamma_t\theta)(\lambda\gamma^t\theta)(\lambda\gamma^n\theta)(\theta\gamma)_{m\alpha} F^{\alpha\beta}(\gamma_n\theta)_\beta\rangle \\ &= g_o'^2 g_c' \frac{2\pi i\alpha'}{9} H_{abc}\langle(\lambda\gamma^m\xi^3)(\xi^2\gamma_t\theta)(\lambda\gamma^t\theta)(\lambda\gamma^n\theta)(\theta\gamma_m\gamma_{abc}\gamma_n\theta)\rangle.\end{aligned}$$

We use here the identity (4.6) two times and the identities of the appendix A in order to obtain

$$\mathcal{A} = -g_o'^2 g_c' \frac{\pi i\alpha'}{34560} H_{abc}\xi^2\gamma^{abc}\xi^3 = g_o'^2 g_c' \frac{\pi i \alpha'}{720}\left(-\frac{1}{48} H_{abc}\xi^2\gamma^{abc}\xi^3\right)$$

and this is the right coupling, proportional to eq. (2.1).

5. Conclusions

In this work we have computed explicitly all correlation functions involving one closed and two open strings in the pure spinor formalism. Comparing with the effective action for the type I supergravity we came to the conclusion that the pure spinor formalism survives one more consistency test and most of the couplings of the effective action were derived here. The mixed string sector of pure spinor has not previously been considered in the literature and there is a lot of research yet to be done. The problems considered here are just the beginning. Higher point amplitudes can be considered and loop corrections to type I supergravity have not been computed from the pure spinor viewpoint.

As discussed in this paper, the pure spinor formalism gives the right coupling between the Kalb-Ramond field and other gauge fields, a result of particular importance in the mixed anomaly cancellation. At tree level, diagrams in which a two form is exchanged between two gauge fields on one side and four on the other side have to be considered for this cancellation. Therefore, a first step may be the computation of the tree-level five point amplitudes involving a Kalb-Ramond field and four gauge bosons. This last idea is left here for future investigations.

Acknowledgments

I would first like to thank my advisor Nathan Berkovits, for useful discussions. I thank Carlos R. Mafra for clarifying many things about amplitude computations. I thank the GAMMA package [8] of Mathematica and I also acknowledge FAPESP grant 04/06639-4 for financial support.

A. Some important identities

During the computations made in this paper, we use extensively the following identities [7]:

$$\langle(\lambda\gamma^a\theta)\left(\lambda\gamma^b\theta\right)(\lambda\gamma^c\theta)(\theta\gamma_{def}\theta)\rangle = \frac{1}{120}\delta^{abc}_{def}, \tag{A.1}$$

$$\langle\left(\lambda\gamma^{abc}\theta\right)(\lambda\gamma_d\theta)(\lambda\gamma_e\theta)(\theta\gamma_{fgh}\theta)\rangle = \frac{1}{70}\delta^{[a}_{[d}\eta_{e][f}\delta^b_g\delta^{c]}_{h]}, \tag{A.2}$$

$$\langle\left(\lambda\gamma^{abcde}\theta\right)(\lambda\gamma_f\theta)(\lambda\gamma_g\theta)(\theta\gamma_{hij}\theta)\rangle = -\frac{1}{42}\delta^{abcde}_{fghij} - \frac{1}{5040}\varepsilon^{abcde}_{fghij}. \tag{A.3}$$

Any other term can be reduced to these above using the identities

$$\gamma^a\gamma^{bc} = \gamma^{abc} + \eta^{ab}\gamma^c - \eta^{ac}\gamma^b, \tag{A.4}$$

$$\gamma^a\gamma^b\gamma^c = \eta^{bc}\gamma^a - \eta^{ac}\gamma^b + \eta^{ab}\gamma^c + \gamma^{abc}, \tag{A.5}$$

$$\begin{aligned}\gamma^a\gamma^{abc}\gamma^d = {} & \eta^{ad}\eta^{ce}\gamma^b - \eta^{ac}\eta^{de}\gamma^b - \eta^{ad}\eta^{be}\gamma^c + \eta^{ab}\eta^{de}\gamma^c \\ & +\eta^{ac}\eta^{be}\gamma^d - \eta^{ab}\eta^{ce}\gamma^d + \eta^{de}\gamma^{abc} - \eta^{ce}\gamma^{abd} + \eta^{be}\gamma^{acd} \\ & -\eta^{ae}\gamma^{bcd} + \eta^{ad}\gamma^{bce} - \eta^{ac}\gamma^{bde} + \eta^{ab}\gamma^{cde} + \gamma^{abcde}\end{aligned} \tag{A.6}$$

and

$$\begin{aligned}\gamma^a\gamma^{abcde}\gamma^f = {} & \eta^{af}\eta^{eg}\gamma^{bcd} - \eta^{ae}\eta^{fg}\gamma^{bcd} - \eta^{af}\eta^{dg}\gamma^{bce} + \eta^{ad}\eta^{fg}\gamma^{bce} \\ & +\eta^{ae}\eta^{dg}\gamma^{bcf} - \eta^{ad}\eta^{eg}\gamma^{bcf} + \eta^{af}\eta^{cg}\gamma^{bde} - \eta^{ac}\eta^{fg}\gamma^{bde} \\ & -\eta^{ae}\eta^{cg}\gamma^{bdf} + \eta^{ac}\eta^{eg}\gamma^{bdf} + \eta^{ad}\eta^{cg}\gamma^{bef} - \eta^{ac}\eta^{dg}\gamma^{bef} \\ & -\eta^{af}\eta^{bg}\gamma^{cde} + \eta^{ab}\eta^{fg}\gamma^{cde} + \eta^{ae}\eta^{bg}\gamma^{cdf} - \eta^{ab}\eta^{eg}\gamma^{cdf} \\ & -\eta^{ad}\eta^{bg}\gamma^{cef} + \eta^{ab}\eta^{dg}\gamma^{cef} + \eta^{ac}\eta^{bg}\gamma^{def} - \eta^{ab}\eta^{cg}\gamma^{def} \\ & +\eta^{fg}\gamma^{abcde} - \eta^{eg}\gamma^{abcdf} + \eta^{dg}\gamma^{abcef} - \eta^{cg}\gamma^{abdef} \\ & +\eta^{bg}\gamma^{acdef} - \eta^{ag}\gamma^{bcdef} + \eta^{af}\gamma^{bcdeg} - \eta^{ae}\gamma^{bcdfg} \\ & +\eta^{ad}\gamma^{bcefg} - \eta^{ac}\gamma^{bdefg} + \eta^{ab}\gamma^{cdefg} + \gamma^{abcdefg},\end{aligned} \tag{A.7}$$

JHEP02(2009)025

B. Gauge invariance

The expression (3.8) must be invariant under all gauge transformations. The first is given by

$$\delta\left(\lambda A^2\right)=Q\Lambda$$

and the variation of the first term in (3.8) is

$$\begin{aligned}\frac{\delta S_1}{\pi i\alpha'} &= \left\langle A^1_m\left(\lambda\tilde{A}^1\right)(Q\Lambda)\left(\lambda\gamma^m W^3\right)\right\rangle \\ &= \left\langle QA^1_m\left(\lambda\tilde{A}^1\right)\Lambda(\lambda\gamma^m W^3)\right\rangle+\left\langle A^1_m Q\left(\lambda\tilde{A}^1\right)\Lambda(\lambda\gamma^m W^3)\right\rangle-\left\langle A^1_m\left(\lambda\tilde{A}^1\right)\Lambda Q(\lambda\gamma^m W^3)\right\rangle \\ &= \left\langle\left[\lambda\gamma_m W+\partial_m\left(\lambda A^1\right)\right]\left(\lambda\tilde{A}^1\right)\Lambda(\lambda\gamma^m W^3)\right\rangle-\frac{1}{4}\left\langle A^1_m\left(\lambda\tilde{A}^1\right)\Lambda((\lambda\gamma^m)_\alpha(\lambda\gamma^{rs})^\alpha F^3_{rs})\right\rangle \\ &= \left\langle\left[\lambda\gamma_m W+\partial_m\left(\lambda A^1\right)\right]\left(\lambda\tilde{A}^1\right)\Lambda(\lambda\gamma^m W^3)\right\rangle-\frac{1}{4}\left\langle A^1_m\left(\lambda\tilde{A}^1\right)\Lambda((\lambda\gamma^m)_\alpha(\lambda\gamma^{rs})^\alpha F^3_{rs})\right\rangle \\ &= \left\langle\partial_m\left(\lambda A^1\right)\left(\lambda\tilde{A}^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle=k^1_m\left\langle\left(\lambda A^1\right)\left(\lambda\tilde{A}^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle.\end{aligned}$$

In the above expression, we have used the pure spinor condition (3.1) and the Fierz identity

$$(\gamma_m)_{(\alpha\beta}(\gamma^m)_{\rho)\sigma}=0.$$

The variation of the second term is

$$\begin{aligned}\frac{\delta S_2}{\pi i\alpha'} &= \left\langle\tilde{A}^1_m\left(\lambda A^1\right)(Q\Lambda)\left(\lambda\gamma^m W^3\right)\right\rangle \\ &= \left\langle Q\tilde{A}^1_m(\lambda A^1)\Lambda(\lambda\gamma^m W^3)\right\rangle+\left\langle\tilde{A}^1_m Q(\lambda A^1)\Lambda(\lambda\gamma^m W^3)\right\rangle-\left\langle\tilde{A}^1_m(\lambda A^1)\Lambda Q(\lambda\gamma^m W^3)\right\rangle \\ &= \left\langle\left[\lambda\gamma_m W^1+\partial_m\left(\lambda\tilde{A}^1\right)\right]\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle-\frac{1}{4}\left\langle\tilde{A}^1_m\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m\lambda\gamma^{rs}F^3_{rs}\right)\right\rangle \\ &= \left\langle\left[\lambda\gamma_m W^1+\partial_m\left(\lambda\tilde{A}^1\right)\right]\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle-\frac{1}{4}\left\langle\tilde{A}^1_m\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m\lambda\gamma^{rs}F^3_{rs}\right)\right\rangle \\ &= \left\langle\partial_m\left(\lambda\tilde{A}^1\right)\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle=k^1_m\left\langle\left(\lambda\tilde{A}^1\right)\left(\lambda A^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle \\ &= -k^1_m\left\langle\left(\lambda A^1\right)\left(\lambda\tilde{A}^1\right)\Lambda\left(\lambda\gamma^m W^3\right)\right\rangle\end{aligned}$$

again we have used the pure spinor condition and the Fierz identity. Adding the results we obtain

$$\delta S=\delta S_1+\delta S_2=0.$$

The other gauge transformation is given by

$$\delta\left(\lambda A^1\right)=Q\Lambda,\delta A^1_m=\partial_m\Lambda$$

and we obtain

$$\frac{\delta S_1}{\pi i\alpha'}=\left\langle\partial_m\Lambda\left(\lambda\tilde{A}^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle=k^1_m\left\langle\Lambda\left(\lambda\tilde{A}^1\right)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle.$$

For the second term

$$\begin{aligned}\frac{\delta S_2}{\pi i\alpha'} &= \left\langle\tilde{A}^1_m(Q\Lambda)\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle \\ &= -\left\langle Q\tilde{A}^1_m\Lambda(\lambda A^2)(\lambda\gamma^m W^3)\right\rangle+\left\langle\tilde{A}^1_m\Lambda Q(\lambda A^2)(\lambda\gamma^m W^3)\right\rangle-\left\langle\tilde{A}^1_m\Lambda(\lambda A^2)Q(\lambda\gamma^m W^3)\right\rangle\end{aligned}$$

$$= -\left\langle \left[\lambda\gamma_m W^1 + \partial_m\left(\lambda\tilde{A}^1\right)\right]\Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle - \frac{1}{4}\left\langle \tilde{A}^1_m \Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m\lambda\gamma^{rs}F^3_{rs}\right)\right\rangle$$
$$= -\left\langle \left[\lambda\gamma_m W^1 + \partial_m\left(\lambda\tilde{A}^1\right)\right]\Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle - \frac{1}{4}\left\langle \tilde{A}^1_m \Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m\lambda\gamma^{rs}F^3_{rs}\right)\right\rangle$$
$$= -\left\langle \partial_m\left(\lambda\tilde{A}^1\right)\Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle = -k^1_m\left\langle \left(\lambda\tilde{A}^1\right)\Lambda\left(\lambda A^2\right)\left(\lambda\gamma^m W^3\right)\right\rangle$$

and finally

$$\delta S = \delta S_1 + \delta S_2 = 0.$$

Using identical arguments as above we obtain the invariance under

$$\delta\lambda\tilde{A} = Q\Lambda.$$

Therefore, as expected, the final expression is in fact gauge invariant.

C. The one graviton two photons correlation function in Ramond-Neveu-Schwarz formalism

In the Ramond-Neveu-Schwarz case two of the vertex operators must be in the picture -1 and one in the picture 0. Choosing the closed string in the picture 0, we obtain

$$V_c^0 = \frac{-2i}{\alpha'}g'_c : c\tilde{c}h_{\mu\nu}\left(i\partial X^\mu + \frac{\alpha\prime}{2}k^1_\sigma\psi^\sigma\psi^\mu\right)\left(i\bar{\partial}X^\nu + \frac{\alpha\prime}{2}k^1_\rho\bar{\psi}^\rho\bar{\psi}^\nu\right)e^{ik^1\cdot x}(z) : .$$

In this section, we follow the notation used in [6].The fixed open string operator in the -1 picture is given by

$$V_o^{-1} = ig'_o : a_{2\alpha}\psi^\alpha c e^{-\phi}e^{ik_2\cdot x}(y_2) :,$$

and the integrated one is given by

$$V_o^{-1} = ig'_o\int dy_3 : a_{3\beta}\psi^\beta e^{-\phi}e^{ik_3\cdot x}(y_3) : .$$

The expression for the amplitude is given by [6]

$$\mathcal{A} = 2i\frac{g'_c}{\alpha\prime}g'^2_o e^{-\lambda}\int_{-\infty}^{+\infty}dy_3\langle : c\tilde{c}h_{\mu\nu}\left(i\partial X^\mu + \frac{\alpha\prime}{2}k^1_\sigma\psi^\sigma\psi^\mu\right)\left(i\bar{\partial}X^\nu + \frac{\alpha\prime}{2}k^1_\rho\bar{\psi}^\rho\bar{\psi}^\nu\right)e^{ik^1\cdot x}(z) : \\ : a_{2\alpha}\psi^\alpha c e^{-\phi}e^{ik_2\cdot x}(y_2) :: a_{3\beta}\psi^\beta e^{-\phi}e^{ik_3\cdot x}(y_3) :\rangle. \quad \text{(C.1)}$$

The OPEs between the X^μ fields will be needed for all cases and it is given by

$$: X^\mu(z_1)X^\nu(z_2) := X^\mu(z_1)X^\nu(z_2) - \frac{\alpha\prime}{2}\eta^{\mu\nu}\left[\ln|z_1 - z_2|^2 + \ln|z_1 - \bar{z}_2|^2\right]. \quad \text{(C.2)}$$

From the above expression all the related OPEs can be obtained

$$: \partial X^\mu(z_1)\bar{\partial}X^\nu(z_2) := \partial X^\mu(z_1)\bar{\partial}X^\nu(z_2) - \frac{\alpha\prime}{2}\eta^{\mu\nu}\frac{1}{(\bar{z}_2 - z_1)^2},$$
$$: \partial X^\mu(z_1)X^\nu(y) := \partial X^\mu(z_1)X^\nu(y) + \alpha\prime\eta^{\mu\nu}\frac{1}{y - z_1},$$
$$: \bar{\partial}X^\mu(z_1)X^\nu(y) := \partial X^\mu(z_1)X^\nu(z_2) + \alpha\prime\eta^{\mu\nu}\frac{1}{y - \bar{z}_1}.$$

We also need of the OPE for the fields

$$\langle e^{-\phi}(z_1)\, e^{-\phi}(z_2)\rangle = z_{12}^{-1},$$
$$\langle \psi^{\mu}(z_1)\, \psi^{\nu}(z_2)\rangle = \eta^{\mu\nu} z_{12}^{-1}.$$

After making all the possible contractions in the expression (C.1), we obtain

$$\begin{aligned}\mathcal{A} = {} & 2i\frac{g\prime_c}{\alpha\prime} g_o^{\prime 2} e^{-\lambda}\int_{-\infty}^{+\infty} dy_3 \langle : c\tilde{c}e^{ik^1\cdot x}(z) :: ce^{-\phi}e^{ik_2\cdot x}(y_2) :: e^{ik_3\cdot x}(y_3) :\rangle \frac{1}{y_2-y_3} \\ & \times\Bigg\{ -h_{\mu\nu}a_{2\alpha}a_{3\beta}\frac{\eta^{\alpha\beta}}{(y_2-y_3)}\left[+\frac{i\alpha\prime}{y_2-z}k_2^{\mu}+\frac{ik_3^{\mu}\alpha\prime}{y_3-z}\right]\left[\frac{i\alpha\prime}{y_2-\overline{z}}k_2^{\nu}+\frac{i\alpha\prime}{y_3-\overline{z}}k_3^{\nu}\right] \\ & +\frac{i\alpha'}{2}h_{\mu\nu}\left[\frac{i\alpha\prime}{y_2-\overline{z}}k_2^{\nu}+\frac{i\alpha\prime}{y_3-\overline{z}}k_3^{\nu}\right]\left[\frac{k_{1\sigma}\eta^{\mu\alpha}\eta^{\sigma\beta}a_{2\alpha}a_{3\beta}}{(z-y_2)(z-y_3)}-\frac{k_{1\sigma}\eta^{\mu\beta}\eta^{\sigma\alpha}a_{2\alpha}a_{3\beta}}{(z-y_2)(z-y_3)}\right] \\ & +\frac{i\alpha'}{2}h_{\mu\nu}\left[+\frac{i\alpha\prime}{y_2-z}k_2^{\mu}+\frac{ik_3^{\mu}\alpha\prime}{y_3-z}\right]\left[\frac{k_{1\rho}\eta^{\nu\alpha}\eta^{\rho\beta}a_{2\alpha}a_{3\beta}}{(\bar{z}-y_2)(\bar{z}-y_3)}-\frac{k_{1\rho}\eta^{\nu\beta}\eta^{\rho\alpha}a_{2\alpha}a_{3\beta}}{(\bar{z}-y_2)(\bar{z}-y_3)}\right]\Bigg\}.\end{aligned} \tag{C.3}$$

The ghost contribution to the amplitude is given by

$$\langle c\widetilde{c}(z)c(z_2)\rangle = C^g_{D_2}\left|y_2-z\right|^2(z-\overline{z}).$$

In the last equation, $C^g_{D_2}$ is a constant coming from functional determinants. The contribution from the exponentials is given by

$$\begin{aligned}&\Big\langle : e^{ik\cdot x}(z) :: e^{ik\cdot x}(y_2) :: e^{ik\cdot x}(y_3) :\Big\rangle \\ &\quad = iC^X_{D_2}(2\pi)^d\delta(\Sigma k)\left|z-\overline{z}\right|^{\alpha\prime k_1^2/2}\left|y_2-y_3\right|^{2\alpha\prime k_2\cdot k_3}\left|y_2-z\right|^{2\alpha\prime k_1\cdot k_2}\left|y_3-z\right|^{2\alpha\prime k_1\cdot k_3}\end{aligned}$$

again, $C^X_{D_2}$ is a constant coming from functional determinants. Using momentum conservation we obtain

$$k_1^2 = k_1\cdot k_2 = k_1\cdot k_3 = k_3\cdot k_2 = 0,$$

then

$$\Big\langle : e^{ik\cdot x}(z) :: e^{ik\cdot x}(y_2) :: e^{ik\cdot x}(y_3) :\Big\rangle = iC^X_{D_2}(2\pi)^d\delta(\Sigma k),$$

and we obtain for (C.3)

$$\begin{aligned}\mathcal{A} = {} & -2\frac{g_c'}{\alpha\prime}g_o^{\prime 2}(2\pi)^d\delta(\Sigma k)e^{-\lambda}C^g_{D_2}C^X_{D_2}\int_{-\infty}^{+\infty}dy_3\frac{1}{y_2-y_3}\left|y_2-z\right|^2(z-\overline{z})(2\pi)^d\delta(\Sigma k) \\ & \times\Bigg\{ -h_{\mu\nu}a_{2\alpha}a_{3\beta}\frac{\eta^{\alpha\beta}}{(y_2-y_3)}\left[+\frac{i\alpha\prime}{y_2-z}k_2^{\mu}+\frac{ik_3^{\mu}\alpha\prime}{y_3-z}\right]\left[\frac{i\alpha\prime}{y_2-\overline{z}}k_2^{\nu}+\frac{i\alpha\prime}{y_3-\overline{z}}k_3^{\nu}\right] \\ & +\frac{i\alpha'}{2}h_{\mu\nu}\left[\frac{i\alpha\prime}{y_2-\overline{z}}k_2^{\nu}+\frac{i\alpha\prime}{y_3-\overline{z}}k_3^{\nu}\right]\left[\frac{k_{1\sigma}\eta^{\mu\alpha}\eta^{\sigma\beta}a_{2\alpha}a_{3\beta}}{(z-y_2)(z-y_3)}-\frac{k_{1\sigma}\eta^{\mu\beta}\eta^{\sigma\alpha}a_{2\alpha}a_{3\beta}}{(z-y_2)(z-y_3)}\right] \\ & +\frac{i\alpha'}{2}h_{\mu\nu}\left[+\frac{i\alpha\prime}{y_2-z}k_2^{\mu}+\frac{ik_3^{\mu}\alpha\prime}{y_3-z}\right]\left[\frac{k_{1\rho}\eta^{\nu\alpha}\eta^{\rho\beta}a_{2\alpha}a_{3\beta}}{(\bar{z}-y_2)(\bar{z}-y_3)}-\frac{k_{1\rho}\eta^{\nu\beta}\eta^{\rho\alpha}a_{2\alpha}a_{3\beta}}{(\bar{z}-y_2)(\bar{z}-y_3)}\right]\Bigg\}.\end{aligned}$$

The contribution of the functional determinants can be found in [6] and it is given by

$$e^{-\lambda}C^g_{D_2}C^X_{D_2} = \frac{1}{\alpha' g_o^2}, g_c' = \frac{2g_c}{\alpha\prime}; g_o' = \frac{g_o}{\sqrt{2\alpha'}}.$$

JHEP02(2009)025

As in the pure spinor case, we fix

$$y_2 = 0; \mathrm{Re}(z) = 0; \mathrm{Im}(z) = a$$

to obtain

$$\mathcal{A} = \frac{-i}{2\alpha'} g'_c (2\pi)^d \delta^d(\Sigma k) a \int_{-\infty}^{+\infty} dy_3 \frac{1}{|y_3 + ia|^2}$$
$$\times \{[a_3 \cdot k_{12} h_{\mu\nu} (a_2^\mu k_{23}^\nu + a_2^\nu k_{23}^\mu) - a_2 \cdot k_{13} h_{\mu\nu} (a_3^\mu k_{23}^\nu + a_3^\nu k_{23}^\mu) + 2h_{\mu\nu} k_{23}^\nu k_{23}^\mu a_2 \cdot a_3]\} .$$

From the above expression, we can already see that the antisymmetric part of $h_{\mu\nu}$ does not contribute for this amplitude. In fact in the type I superstring, the Kalb-Ramond contribution comes from the RR sector and not from NS-NS. Finally, integrating we obtain

$$\mathcal{A} = \frac{\pi i}{2\alpha\prime} g_c (2\pi)^d \delta^d(\Sigma k) \left[a_3 \cdot k_{12} h_{\mu\nu} a_2^\mu k_{23}^\nu - a_2 \cdot k_{13} h_{\mu\nu} a_3^\mu k_{23}^\nu + h_{\mu\nu} k_{23}^\nu k_{23}^\mu a_2 \cdot a_3\right] .$$

The last expression can be written in the position space

$$\mathcal{A} = \frac{i}{4\alpha\prime} g_c h_\nu^\mu F_{\mu\alpha} F^{\nu\alpha} . \tag{C.4}$$

This amplitude originates a term in the effective action that is proportional to (2.1) and to the pure spino result (4.3),as desired. Obviously it has all the desired properties as gauge invariance and symmetry in the exchange of the two photons.

D. One gravitino one photon one photino

As said in the text, the final expression is given by

$$\begin{aligned}
\mathcal{A} = & -\frac{\pi i \alpha'}{8} \langle \left[(\lambda\gamma^{g_1}\theta)(\psi_{g_1}\gamma_m\theta) + \frac{2}{3}(\psi_m\gamma_r\theta)(\lambda\gamma^r\theta) \right] a_{f_2}^2 \left(\lambda\gamma^{f_2}\theta\right) (\lambda\gamma_m\gamma^{m_3 s}\theta) \partial_{m_3}\xi^3\gamma_s\theta \rangle \\
& -\frac{\pi i \alpha'}{12} \langle \left[(\lambda\gamma^{g_1}\theta)(\psi_{g_1}\gamma_m\theta) + \frac{2}{3}(\psi_m\gamma_r\theta)(\lambda\gamma^r\theta) \right] (\xi^2\gamma_t\theta)(\lambda\gamma^t\theta) \left(\lambda\gamma_m\gamma^{m_3 f_3}\theta\right) F_{m_3 f_3}^3 \rangle \\
& +\frac{\pi i \alpha'}{32} \langle \left[(\lambda\gamma^{g_1}\theta)(\psi_{g_1}\gamma_m\theta) + \frac{2}{3}(\psi_m\gamma_r\theta)(\lambda\gamma^r\theta) \right] F_{m_2 f_2}^2 (\lambda\gamma_p\theta) \left(\theta\gamma^{m_2 f_2 p}\theta\right) (\lambda\gamma_m\xi^3) \rangle \\
& +\pi i \alpha' \langle \left[\frac{1}{48} (\lambda\gamma^{g_1}\theta)(\theta\gamma_m\gamma^{m_1 q}\theta)(\partial_{m_1}\psi_{g_1}\gamma_q\theta) a_{f_2}^2 \left(\lambda\gamma^{f_2}\theta\right)(\lambda\gamma_m\xi^3) \right. \\
& \qquad + \frac{1}{24} (\partial_{m_1}\psi_{g_1}\gamma_r\theta)(\lambda\gamma^r\theta)(\theta\gamma_m\gamma^{m_1 g_1}\theta) a_{f_2}^2 \left(\lambda\gamma^{f_2}\theta\right)(\lambda\gamma_m\xi^3) \\
& \qquad + \frac{1}{32} (\lambda\gamma_t\theta)\left(\theta\gamma^{m_1 g_1 t}\theta\right)(\partial_{m_1}\psi_{g_1}\gamma_m\theta) a_{f_2}^2 \left(\lambda\gamma^{f_2}\theta\right)(\lambda\gamma_m\xi^3) \\
& \qquad \left. + \frac{1}{120} (\lambda\gamma_r\theta)\left(\theta\gamma^{rg1t}\theta\right)(\partial_m\psi_{g_1}\gamma_t\theta) a_{f_2}^2 \left(\lambda\gamma^{f_2}\theta\right)(\lambda\gamma_m\xi^3) \right] \rangle
\end{aligned}$$

the terms with five θ's are given by

$$\begin{aligned}
\mathcal{A} = & +\frac{\pi i \alpha'}{8} a_{f_2}^2 \langle (\lambda\gamma_r\gamma^{m_3 s}\theta)(\lambda\gamma^{g_1}\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma^r\psi_{g_1}) \partial_{m_3}\xi^3\gamma_s\theta \rangle \\
& +\frac{\pi i \alpha'}{12} a_{f_2}^2 \langle (\lambda\gamma^{g_1}\gamma^{m_3 s}\theta)(\lambda\gamma^r\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma_r\psi_{g_1}) \partial_{m_3}\xi^3\gamma_s\theta \rangle
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\pi i\alpha'}{12}F^3_{m_3f_3}\langle\left(\lambda\gamma^r\gamma^{m_3f_3}\theta\right)(\lambda\gamma^{g_1}\theta)\left(\lambda\gamma^t\theta\right)(\theta\gamma_r\psi_{g_1})\left(\xi^2\gamma_t\theta\right)\rangle\\
&+\frac{\pi i\alpha'}{18}F^3_{m_3f_3}\langle\left(\lambda\gamma^{g_1}\gamma^{m_3f_3}\theta\right)(\lambda\gamma^r\theta)\left(\lambda\gamma^t\theta\right)(\theta\gamma_r\psi_{g_1})\left(\xi^2\gamma_t\theta\right)\rangle\\
&+\frac{\pi i\alpha'}{32}F^2_{m_2f_2}\langle\left(\lambda\gamma_r\xi^3\right)(\psi_{g_1}\gamma^r\theta)(\lambda\gamma^{g_1}\theta)(\lambda\gamma_p\theta)\left(\theta\gamma^{m_2f_2p}\theta\right)\rangle\\
&+\frac{\pi i\alpha'}{48}F^2_{m_2f_2}\langle\left(\lambda\gamma^{g_1}\xi^3\right)(\psi_{g_1}\gamma_r\theta)(\lambda\gamma^r\theta)(\lambda\gamma_p\theta)\left(\theta\gamma^{m_2f_2p}\theta\right)\rangle\\
&+\frac{\pi i\alpha'}{48}a^2_{f_2}\langle\left(\lambda\gamma_p\xi^3\right)(\partial_{m_1}\psi_{g_1}\gamma_q\theta)(\lambda\gamma^{g_1}\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma^p\gamma^{m_1q}\theta)\rangle\\
&+\frac{\pi i\alpha'}{24}a^2_{f_2}\langle\left(\lambda\gamma_s\xi^3\right)(\partial_{m_1}\psi_{g_1}\gamma_r\theta)(\lambda\gamma^r\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma^s\gamma^{m_1g_1}\theta)\rangle\\
&+\frac{\pi i\alpha'}{32}a^2_{f_2}\langle\left(\lambda\gamma_s\xi^3\right)(\partial_{m_1}\psi_{g_1}\gamma^s\theta)(\lambda\gamma_t\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma^{m_1g_1t}\theta)\rangle\\
&+\frac{\pi i\alpha'}{120}a^2_{f_2}\langle\left(\lambda\gamma^{m_1}\xi^3\right)(\partial_{m_1}\psi_{g_1}\gamma_t\theta)(\lambda\gamma_r\theta)\left(\lambda\gamma^{f_2}\theta\right)(\theta\gamma^{rg_1t}\theta)\rangle
\end{aligned}
$$

all the terms above are quite similar to the one graviton and two photinos computation. Following the same steps and using the fact that the gravitino is gamma traceless extensively we obtain

$$
\mathcal{A}_1 = \pi i\alpha'\left(\frac{1}{3840}a_{f_2}\psi_{g_1}\gamma^{f_2}\partial^{g_1}\xi_3 - \frac{1}{7680}a_{f_2}\psi_{g_1}\gamma^{f_2g_1m_3}\partial_{m_3}\xi_3\right).
$$

Using now the identity (A.5), we obtain

$$
\begin{aligned}
\mathcal{A}_1 &= \pi i\alpha'\left(\frac{1}{3840}a_{f_2}\psi_{g_1}\gamma^{f_2}\partial^{g_1}\xi_3 + \frac{1}{7680}a_{f_2}\psi_{g_1}\gamma^{f_2}\partial^{g_1}\xi_3\right)\\
&= \frac{3\pi i\alpha'}{7680}a_{f_2}\psi_{g_1}\gamma^{f_2}\partial^{g_1}\xi_3.
\end{aligned}
$$

Using the same argument for the other ten terms, we obtain

$$
\begin{aligned}
\mathcal{A}_2 &= \frac{\pi i\alpha'}{2160}a_{f_2}\psi_{g_1}\gamma^{f_2}\partial^{g_1}\xi_3,\\
\mathcal{A}_3 &= \pi i\alpha'\left(-\frac{1}{5760}F^3_{m_3f_3}\psi^{m_3}\gamma^{f_3}\xi_2 + \frac{1}{5760}F^3_{m_3f_3}\psi^{f_3}\gamma^{m_3}\xi_2 + \frac{1}{5760}F^3_{m_3f_3}\psi_{g_1}\gamma^{f_3g_1m_3}\xi_2\right)\\
&= -\frac{\pi i\alpha'}{1440}F^3_{m_3f_3}\psi^{m_3}\gamma^{f_3}\xi_2,\\
\mathcal{A}_4 &= -\frac{\pi i\alpha'}{2880}F^3_{m_3f_3}\psi^{m_3}\gamma^{f_3}\xi_2 + \frac{\pi i\alpha'}{2880}F^3_{m_3f_3}\psi^{f_3}\gamma^{m_3}\xi_2\\
&= -\frac{\pi i\alpha'}{1440}F^3_{m_3f_3}\psi^{m_3}\gamma^{f_3}\xi_2,\\
\mathcal{A}_5 &= \pi i\alpha'\left(\frac{1}{5760}F^2_{m_2f_2}\xi^3\gamma^{f_2}\psi^{m_2} - \frac{1}{5760}F^2_{m_2f_2}\xi^3\gamma^{m_2}\psi^{f_2} - \frac{1}{46080}F^2_{m_2f_2}\xi^3\gamma^{f_2g_1m_2}\psi_{g_1}\right)\\
&= \pi i\alpha'\left(\frac{1}{2880}F^2_{m_2f_2}\xi^3\gamma^{f_2}\psi^{m_2} - \frac{1}{23040}F^2_{m_2f_2}\xi^3\gamma^{f_2}\psi^{m_2}\right) = \frac{7\pi i\alpha'}{23040}F^2_{m_2f_2}\xi^3\gamma^{f_2}\psi^{m_2},
\end{aligned}
$$

$$\begin{aligned}
\mathcal{A}_6 &= \pi i \alpha' \left(\frac{1}{17280} F^2_{m_2 f_2} \xi^3 \gamma^{f_2} \psi^{m_2} - \frac{1}{17280} F^2_{m_2 f_2} \xi^3 \gamma^{m_2} \psi^{f_2} + \frac{1}{17280} F^2_{m_2 f_2} \xi^3 \gamma^{f_2 g_1 m_2} \psi_{g_1} \right) \\
&= \pi i \alpha' \left(\frac{1}{8640} F^2_{m_2 f_2} \xi^3 \gamma^{f_2} \psi^{m_2} + \frac{1}{8640} F^2_{m_2 f_2} \xi^3 \gamma^{f_2} \psi^{m_2} \right) = \frac{\pi i \alpha'}{4320} F^2_{m_2 f_2} \xi^3 \gamma^{f_2} \psi^{m_2}, \\
\mathcal{A}_7 &= \pi i \alpha' \left(\frac{1}{34560} a^2_{m_1} \xi^3 \gamma^{g_1} \partial^{m_1} \psi_{g_1} - \frac{1}{138240} a^{f_2}_2 \xi^3 \gamma_{f_2 g_1 m_1} \partial^{m_1} \psi^{g_1} \right) \\
&= 0, \\
\mathcal{A}_8 &= -\frac{\pi i \alpha'}{17280} a^2_{f_2} \xi^3 \gamma^{f_2 g_1 m_1} \partial_{m_1} \psi_{g_1} = 0, \\
\mathcal{A}_9 &= -\frac{\pi i \alpha'}{46080} a^2_{f_2} \xi^3 \gamma^{f_2 g_1 m_1} \partial_{m_1} \psi_{g_1} = 0
\end{aligned}$$

and

$$\mathcal{A}_{10} = \pi i \alpha' \left(\frac{1}{1440} a^2_{f_2} \xi^3 \gamma^{f_2 g_1 m_1} \partial_{m_1} \psi_{g_1} \right) = 0.$$

We can note that the last four terms give null results because, as in the graviton case, there is no way to contract a kinetic term of the gravitino with a photon and a photino that gives a non null result. Adding all results, we obtain

$$\begin{aligned}
\mathcal{A} = \pi i \alpha' \bigg(&- \frac{3}{7680} F^2_{g_1 f_2} \psi_{g_1} \gamma^{f_2} \xi_3 - \frac{1}{2160} F^2_{g_1 f_2} \psi_{g_1} \gamma^{f_2} \xi_3 \\
&- \frac{1}{1440} F^3_{m_3 f_3} \psi^{m_3} \gamma^{f_3} \xi_2 - \frac{1}{1440} F^3_{m_3 f_3} \psi^{m_3} \gamma^{f_3} \xi_2 \\
&- \frac{7}{23040} F^2_{m_2 f_2} \psi^{m_2} \gamma^{f_2} \xi^3 - \frac{1}{4320} F^2_{m_2 f_2} \psi^{m_2} \gamma^{f_2} \xi^3 \bigg) \\
= -\frac{\pi i \alpha'}{720} &(F^2_{g_1 f_2} \psi^{g_1} \gamma^{f_2} \xi_3 + F^3_{m_3 f_3} \psi^{m_3} \gamma^{f_3} \xi_2)
\end{aligned}$$

References

[1] N. Berkovits, *Super-Poincaré covariant quantization of the superstring*, *JHEP* **04** (2000) 018 [hep-th/0001035].

[2] C.R. Mafra, *Four-point one-loop amplitude computation in the pure spinor formalism*, *JHEP* **01** (2006) 075 [hep-th/0512052].

[3] N. Berkovits and C.R. Mafra, *Equivalence of two-loop superstring amplitudes in the pure spinor and RNS formalisms*, *Phys. Rev. Lett.* **96** (2006) 011602 [hep-th/0509234].

[4] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, *Ten-dimensional Maxwell-Einstein supergravity, its currents and the issue of its auxiliary fields*, *Nucl. Phys.* **B 195** (1982) 97.

[5] C.R. Mafra, *Pure spinor superspace identities for massless four-point kinematic factors*, *JHEP* **04** (2008) 093 [arXiv:0801.0580].

[6] J. Polchinski, *String theory. Vol. 1: an introduction to the bosonic string*, Cambridge University Press, Cambridge, U.K. (1998).

[7] N. Berkovits and C.R. Mafra, *Some superstring amplitude computations with the non-minimal pure spinor formalism*, *JHEP* **11** (2006) 079 [hep-th/0607187].

[8] U. Gran, *GAMMA: a Mathematica package for performing Gamma-matrix algebra and Fierz transformations in arbitrary dimensions*, hep-th/0105086.